

-Evolution Equation of Global Surface Temperature (correction 4) '09/11/3, 5, 17

<<EGT solution by "step by step integration with $\alpha(t)$ ">>

In the "Global Temperature Fact", the linear leading temperature $T_A(t)$ was assumed. (<http://www.geocities.jp/sqkh5981g/Global-temperature-fact.pdf>). In the below, direct EGT solution by "step by step integration with $\alpha(t)$ " is shown.

[1] : The sensitivity of $RF = \delta F_0$ with regard to $\{m, \alpha\}$.

In the below, $\{m(t), \alpha(t)\}$ are to be estimated concretely to fit geo-data in 1750 to 2008. And then, those are to become **policy variables** for $T_G(t)$ down by pulling down of CO2 concentration (385ppm \rightarrow (385-1.5y)ppm) as "a possible simulation".

Then $\{m(t)=\text{constant}, \alpha(t)\}$ is assumed. Thus problem are

- (a) deriving current $\alpha(t=2008)$ and $m(t=2008)$,
- (b) deriving year change $\alpha(t)$ by 1.5ppm(max value ?) CO2 pulling down,
- (c) calculating integration of the EGT solution as an approximated one.

(1) surplus heat flow at top of atmosphere (RF) :

$$\{(F_0/4) [1-m(t)] - \alpha(t) \sigma T_G(t)^4\} = \delta F_0 = \delta F_G = 1.6 \text{ W/m}^2. \text{ (IPCC)}$$

$$\begin{aligned} * C_G (dT_G/dt) &= 4 \pi R_E^2 \delta F_G = 4 \pi R_E^2 \sigma \{(F_0/4 \sigma) [1-m(t)] - \alpha(t) T_G(t)^4\}. \\ &= 4 \pi R_E^2 \alpha(t) \sigma \{(F_0/4 \sigma) [1-m(t)] / \alpha(t) - T_G(t)^4\} \\ &= 4 \pi R_E^2 \sigma \{(F_0/4 \sigma) [1-m(t)] + (1-\alpha(t)) T_G(t)^4 - T_G(t)^4\}. \end{aligned}$$

$$(2) dT_G/dt = (4 \pi R_E^2 \sigma / C_G) \{(F_0/4 \sigma) [1-m(t)] - \alpha(t) T_G(t)^4\}.$$

$$K_G = (4 \pi R_E^2 \sigma / C_G).$$

At this time, above EGT formulation=(2) shall be adopted.

$$(3) \partial (\delta F_0) / \partial \alpha(t) = \sigma T_G(t)^4: \text{The 1750 CR intensity} = 383 \text{ W/m}^2. (387 / (2008)).$$

$$(4) \partial (\delta F_0) / \partial m(t) = (F_0/4): \text{The current SR intensity} = 342 \text{ W/m}^2:$$

(5) CO2 concentration change since 1750 to 2008.

$$385 \text{ ppm} (2008) - 280 \text{ ppm} (1750) = 105 \text{ ppm}.$$

(6) $\alpha(t)$ sensitivity with GHG concentration=CO2.

$$\delta F_0(\text{CO2}) = F_0(385 \text{ ppm}; 2008) - F_0(280 \text{ ppm}; 1750) = 1.6 \text{ W/m}^2. \text{ (IPCC)}.$$

(7)albedo and @ppm with 286.7K in 1750:

{m & @} could determine with each other in following relations.

$$\text{heat input (net solar ray (SR))} = \text{heat output (net blackbody cooling radiation (CR))}.$$

$$[1-m] (F_0/4) = @ \sigma T_G^4. \rightarrow @ = [1-m] (F_0/4) / \sigma T_G^4.$$

$$@ = [1-m] (F_0/4) / \sigma T_G^4 = [1-m] \times 0.8914 \text{ in steady balanced state.}$$

$$m = 1 - @ \sigma T_G^4 / (F_0/4).$$

RF change by {m, @} :

m=0.29? (1750)				
@=0.617 (1750)				

(8)albedo and @ppm with 287.5K in 2008:

$$\text{Surplus heat input (radiative forcing)} = [\text{net SR input}] - \text{net CR output}].$$

$$\delta F_0 = [1-m] (F_0/4) - @ \sigma T_G^4. \rightarrow @ = \{ [1-m] (F_0/4) - \delta F_0 \} / \sigma T_G^4.$$

$$@ = \{ [1-m] (F_0/4) - \delta F_0 \} / \sigma T_G^4 = [1-m] \times 0.8816 - 0.0041. \text{ in non-balanced state.}$$

RF change by {m, @} : $\Delta F_0 = \Delta @ \cdot \partial (\delta F_0) / \partial @ (t)$; $\Delta F_0 = \Delta m \cdot \partial (\delta F_0) / \partial m (t)$;

m=0.3! (2008)				
@=0.613 (2008)				
$\Delta @ = -0.0042$				
$\Delta F_0 = 1.6W$				

$$\rightarrow (2) \partial (\delta F_0) / \partial @ (t) = \sigma T_G (t=1750)^4 = 383W/m^2. \rightarrow 383W. \Delta @ = 1.6W; \rightarrow \Delta @ = -0.0042.$$

(9)- $\Delta @ \cdot \sigma T_G (t)^4 = \Delta F \approx 5.35 \times \ln(C/C_0)$ IPCC(1990) and Myhre et al.

The right term is "decisive "for calculating RF(ΔF) by GHG concentration change { $C_0 \rightarrow C$ }. (*author have not yet read the original papers).

(10) deriving @ (t) :

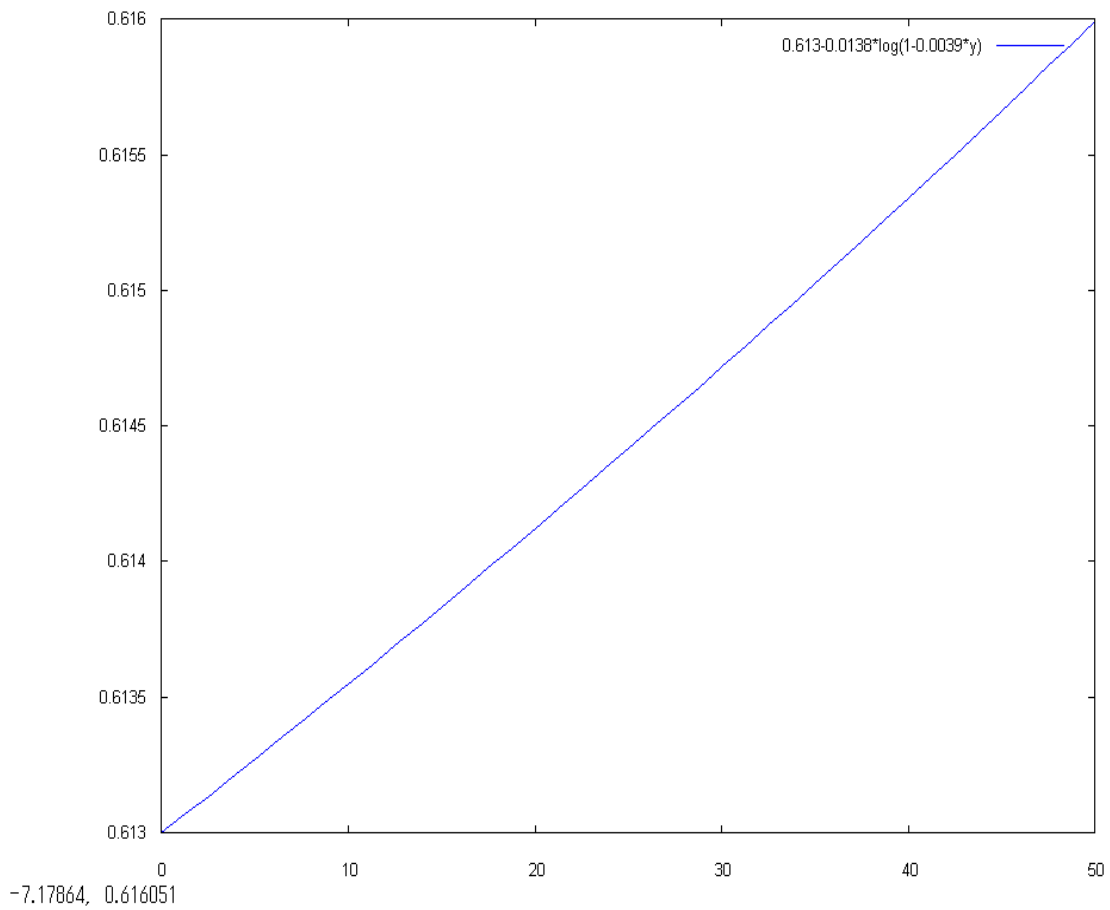
—@ppm variation by CO2 concentration—:

$$\Delta @ (t) \doteq \equiv -5.35 \times \ln \langle C(t) / C(t_0) \rangle / \sigma \langle T_G(t) \rangle^4.$$

$$@ (t) = @_0 (t_0=2008) + \Delta @ (t) = 0.613 - (5.35/387) \times \ln \langle C(t) / 385 \text{ppm} (t_0) \rangle$$

$$= 0.61 - (0.0138) \times \ln \langle (385 - 1.5y) \text{ppm} / 385 \text{ppm} \rangle \doteq 0.613 + 6 \times 10^{-5} y.$$

:plot2d(0.613-0.0138*log(1-0.0039*y), [y, 0, 50]);



(1) The explicit functional of $T_A(\theta(t))$ with assuming constant m (albedo).

$$T_A(t)^4 \equiv (F_0/4\sigma) [1-m(t)] + (1-\theta(t)) [T_G(t)^4].$$

$$T_A(t) = \{(F_0/4\sigma) [1-m(t)] + (1-\theta(t)) [T_G(t)^4]\}^{1/4}.$$

$$T_A(t) = T_A(t_0) + dt \cdot dT_A(t)/dt.$$

$$dT_A(t)/dt = (\partial T_A / \partial \theta) (d\theta(t)/dt)$$

$$= (1/4) \{(F_0/4\sigma) [1-m(t)] + (1-\theta(t)) [T_G(t)^4]\}^{-3/4} \times \langle -[T_G(t)^4] \rangle \langle d\theta(t)/dt \rangle$$

$$= (1/4) \{ \{(F_0/4\sigma) [1-m(t)] + (1-\theta(t)) [T_G(t)^4]\}^{1/4} / [T_A(t)^4] \} \langle -[T_G(t)^4] \rangle \langle d\theta(t)/dt \rangle$$

$$dT_A(t)/dt = (-1/4) \langle d\theta(t)/dt \rangle T_A(t) \langle T_G(t) / T_A(t) \rangle^4 \doteq (-1/4) \langle d\theta(t)/dt \rangle T_A(t).$$

$$T_A(t) \doteq T_A(0) \exp[-(1/4) \langle \theta(t) - \theta(0) \rangle] \doteq T_A(0) [1 - 1/4 \langle \theta(t) - \theta(0) \rangle].$$

$$\sigma T_A(t)^4 \equiv (F_0/4) [1-m(t)] / \theta(t)$$

$$* (10) \rightarrow \theta(t) = \theta_0(t_0=2008) + \Delta\theta(t) = 0.613 - (5.35/387) \times \ln \langle C(t) / 385 \text{ppm}(t_0) \rangle$$

$$= 0.613 - (0.0138) \times \ln \langle (385 - 1.5y) \text{ppm} / 385 \text{ppm} \rangle \doteq 0.613 + 6 \times 10^{-5} y.$$

$$T_A(t) \doteq T_A(0) [1 - 1.25 \times 10^{-5} y].$$

$$\Delta F = 5.35 * \ln(C/C_0=280 \text{ppm}).$$

[2] : Deriving approximated EGT solution by varing $\{ \phi \}$, but not by $T_A(t)$.

(1) $dT_G/dt = K_G \{ (F_0/4 \sigma) [1-m(t)] - \phi(t) T_G(t)^4 \}$.

(2) $K_G \equiv (4 \pi R_E^2 \sigma / C_G) \times 3600 \times 24 \times 365 = 7.09 \times 10^{-10}$.

(3) $\phi(t=2008) = 0.613$ (385ppm) ; $\phi(t=1750) = 0.617$ (280ppm).

policy variable:

$\phi(t) \equiv 0.613 + 0.004(1.5y/105)$. <linear approximation with 1.5ppm/year down>

(4) $T_G(t) = 287.5K$.

(5) $F \equiv (F_0/4 \sigma) [1-m(t)] = 1366(1-0.3) / (4 \times 5.67 \times 10^{-8}) = 4.2160 \times 10^9$.

(6) solution algorithm by step by step integration with $\Delta t=1$ year.

0	$\phi(t)$	$T_G(t \equiv t_0)$
1	$\phi(t + \Delta t)$	$T_G(t + \Delta t) = T_G(t) + \Delta t (dT_G/dt) = T_G(t) + \Delta t K_G \{ F - \phi(t) T_G(t)^4 \}$
n	$\phi(t + n \Delta t)$	$T_G(t + n \Delta t) = \dots$
n+1		$T_G(t + (n+1) \Delta t) = T_G(t + n \Delta t) + \Delta t K_G \{ F - \phi(t + n \Delta t) T_G(t + n \Delta t)^4 \}$

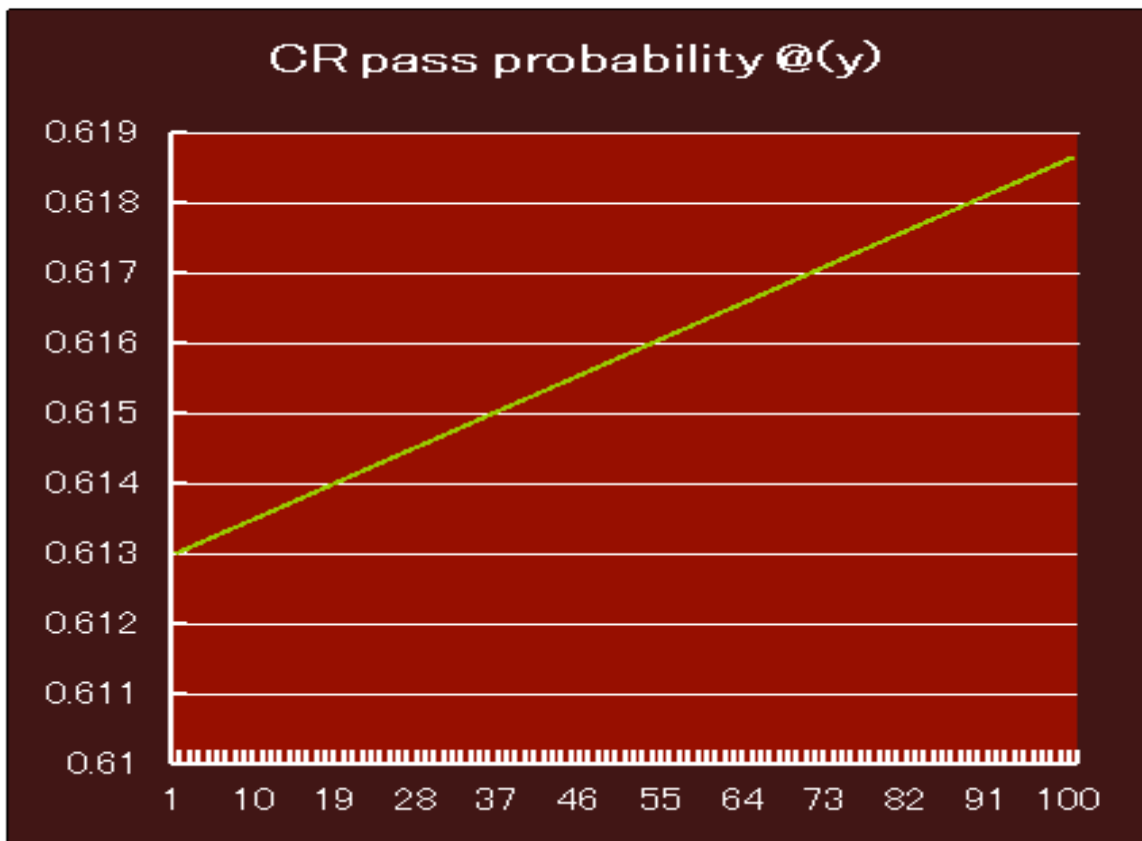
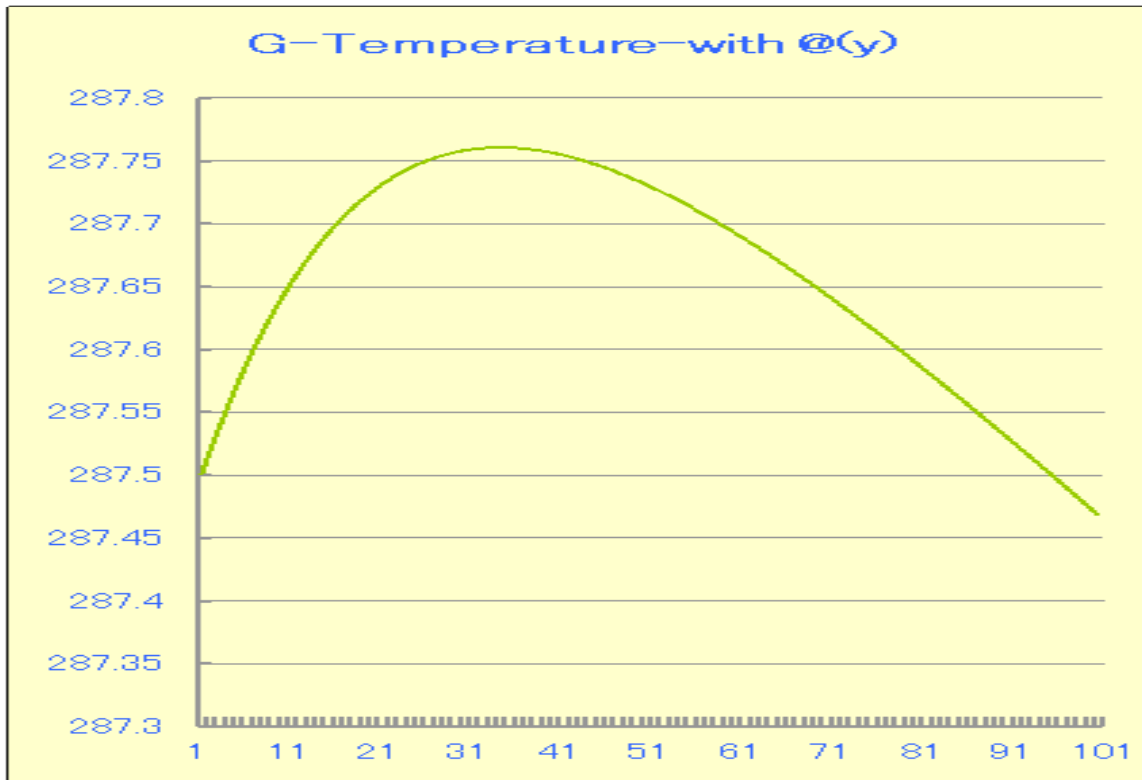
(7) functions in Spreadsheet:

(a) $A[N+1] = A[N] + 5.7143 \times 10^{-4} \dots \dots \dots \langle \phi(t + (n+1) \Delta t) \rangle$.

(b) $B[N+1] = B[N] + K(F - A[N] * (B[N])^4) \dots \dots \dots \langle T_G(t + (n+1) \Delta t) \rangle$.

note: The remarkable difference between the method $\{ T_A(t), \phi(t) \}$ is time scale.

(8)output result:



0 years	@(y)=0. 613	$T_G(y)=287.5$
1	0. 613057143	287. 51981646
2	0. 613114286	287. 5385373
3	0. 613171429	287. 55620749
4	0. 613228572	287. 57287017
5	0. 613285715	287. 58856674
6	0. 613342858	287. 60333692
7	0. 613400001	287. 61721882
8	0. 613457144	287. 63024899
9	0. 613514287	287. 64246251
10	0. 61357143	287. 65389302
11	0. 613628573	287. 6645728
12	0. 613685716	287. 67453279
13	0. 613742859	287. 68380268
14	0. 613800002	287. 69241097
15	0. 613857145	287. 70038496
16	0. 613914288	287. 70775085
17	0. 613971431	287. 71453376
18	0. 614028574	287. 7207578
19	0. 614085717	287. 72644606
20	0. 61414286	287. 73162072
21	0. 614200003	287. 73630301
22	0. 614257146	287. 74051333
23	0. 614314289	287. 74427121
24	0. 614371432	287. 7475954
25	0. 614428575	287. 75050385
26	0. 614485718	287. 75301381
27	0. 614542861	287. 75514179
28	0. 614600004	287. 75690362
29	0. 614657147	287. 75831451
30	0. 61471429	287. 75938901
31	0. 614771433	287. 76014108
32	0. 614828576	287. 7605841
33	0. 614885719	287. 76073093
34	0. 614942862	287. 76059385
35	0. 615000005	287. 76018466

36	0. 615057148	287. 75951468
37	0. 615114291	287. 75859474
38	0. 615171434	287. 75743525
39	0. 615228577	287. 75604616
40	0. 61528572	287. 75443704
41	0. 615342863	287. 75261703
42	0. 615400006	287. 75059492
43	0. 615457149	287. 74837913
44	0. 615514292	287. 74597772
45	0. 615571435	287. 74339842
46	0. 615628578	287. 74064866
47	0. 615685721	287. 73773554
48	0. 615742864	287. 73466587
49	0. 615800007	287. 73144619
50	0. 61585715	287. 72808275
51	0. 615914293	287. 72458155
52	0. 615971436	287. 72094836
53	0. 616028579	287. 71718867
54	0. 616085722	287. 71330778
55	0. 616142865	287. 70931076
56	0. 616200008	287. 70520246
57	0. 616257151	287. 70098753
58	0. 616314294	287. 69667044
59	0. 616371437	287. 69225546
60	0. 61642858	287. 6877467
61	0. 616485723	287. 68314808
62	0. 616542866	287. 67846337
63	0. 616600009	287. 67369619
64	0. 616657152	287. 66884998
65	0. 616714295	287. 66392807
66	0. 616771438	287. 65893363
67	0. 616828581	287. 65386972
68	0. 616885724	287. 64873925
69	0. 616942867	287. 64354502
70	0. 61700001	287. 63828972
71	0. 617057153	287. 63297592

72	0. 617114296	287. 62760608
73	0. 617171439	287. 62218257
74	0. 617228582	287. 61670764
75	0. 617285725	287. 61118347
76	0. 617342868	287. 60561214
77	0. 617400011	287. 59999565
78	0. 617457154	287. 59433589
79	0. 617514297	287. 58863472
80	0. 61757144	287. 58289387
81	0. 617628583	287. 57711503
82	0. 617685726	287. 57129982
83	0. 617742869	287. 56544979
84	0. 617800012	287. 5595664
85	0. 617857155	287. 55365108
86	0. 617914298	287. 5477052
87	0. 617971441	287. 54173005
88	0. 618028584	287. 53572688
89	0. 618085727	287. 5296969
90	0. 61814287	287. 52364125
91	0. 618200013	287. 51756104
92	0. 618257156	287. 5114573
93	0. 618314299	287. 50533107
94	0. 618371442	287. 4991833
95	0. 618428585	287. 49301492
96	0. 618485728	287. 48682683
97	0. 618542871	287. 48061987
98	0. 618600014	287. 47439487
99	0. 618657157	287. 4681526